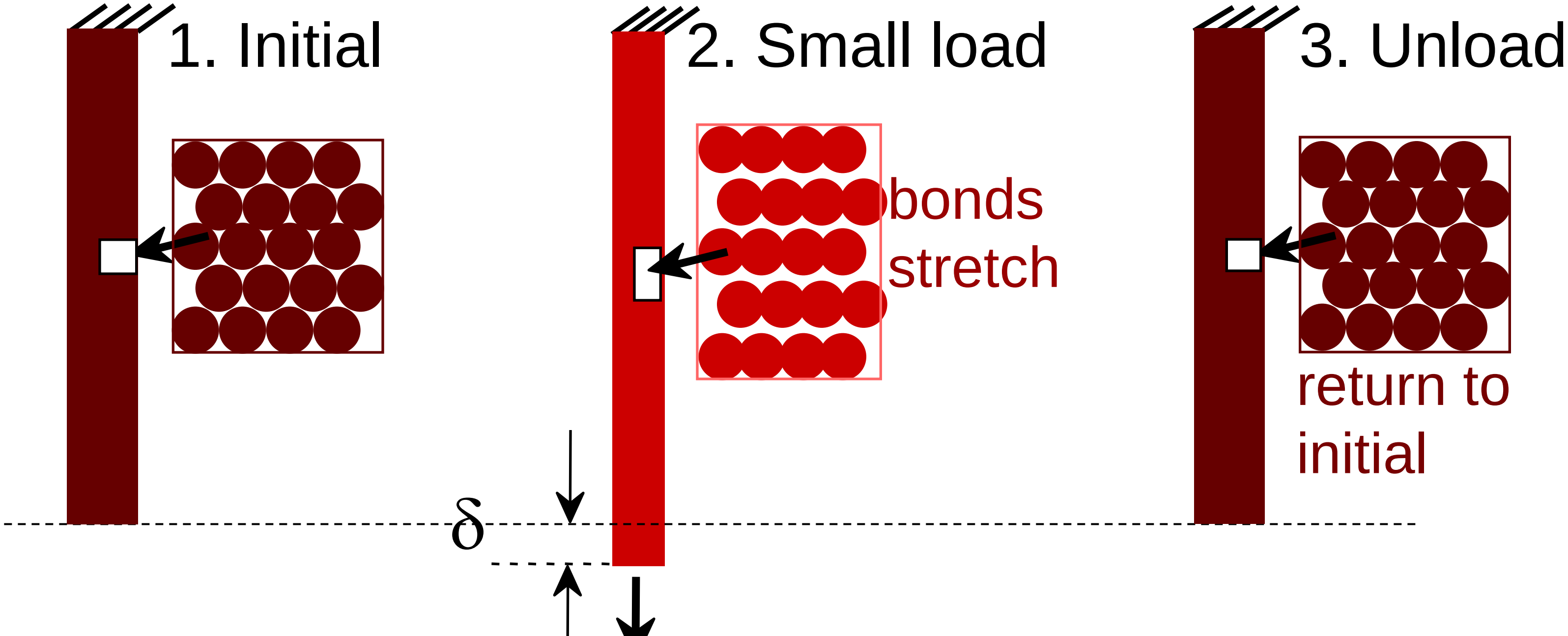
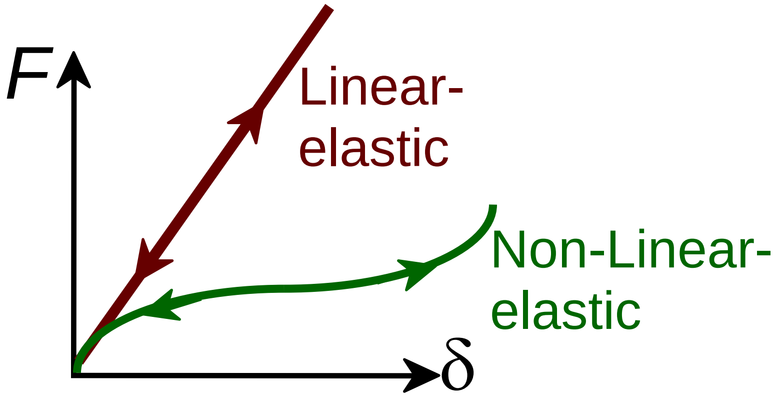
Chapter 6:
Mechanical Properties

ISSUES TO ADDRESS...

- Stress and strain: What are they and why are they used instead of load and deformation?
- Elastic behavior: When loads are small, how much deformation occurs? What materials deform least?
- Plastic behavior: At what point does permanent deformation occur? What materials are most resistant to permanent deformation?
- Toughness and ductility: What are they and how do we measure them?

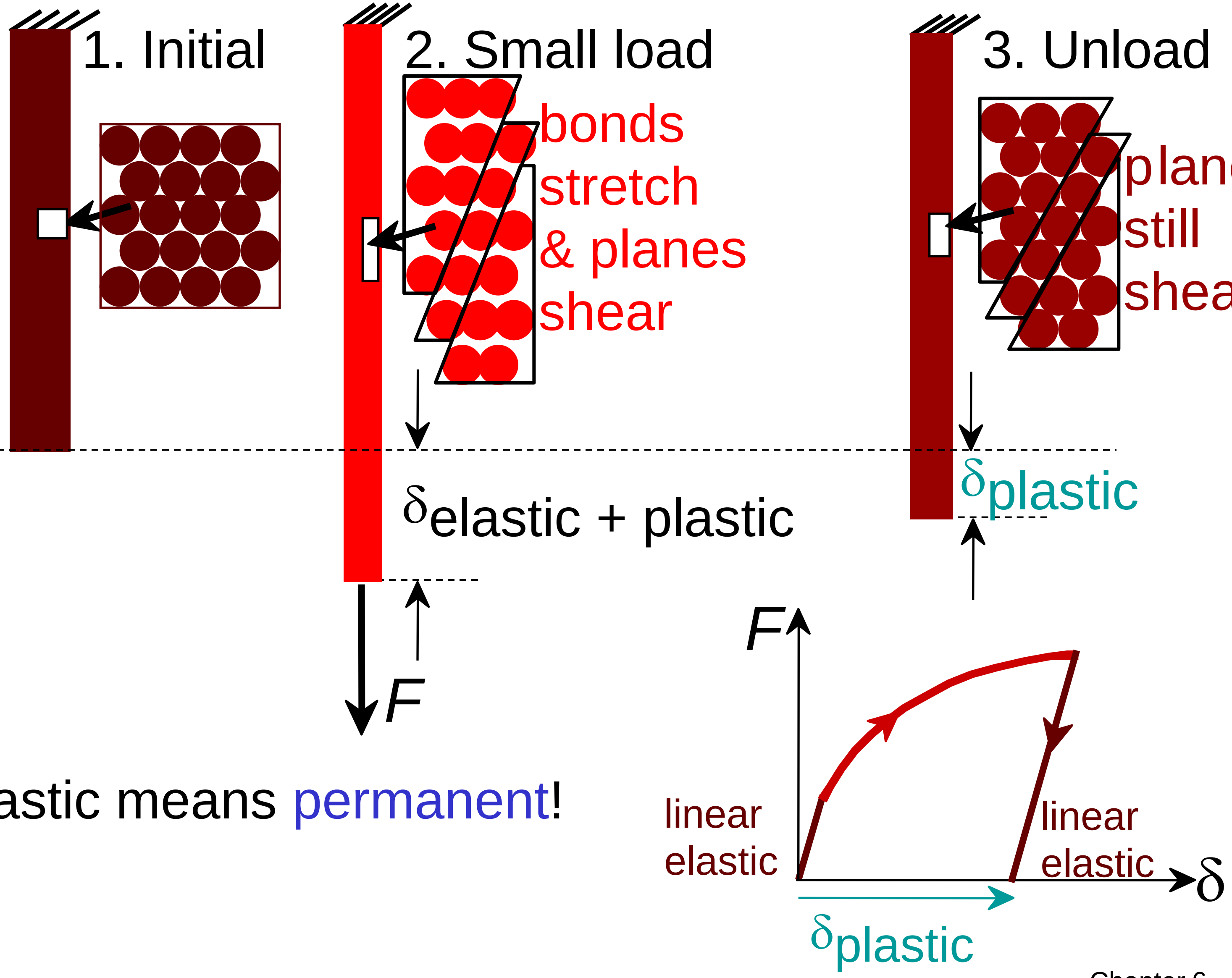
Elastic Deformation





Elastic means reversible!

Plastic Deformation (Metals)



1. Initial

2. Small load

bonds
stretch
& planes
shear

$\delta_{\text{elastic + plastic}}$

F

3. Unload

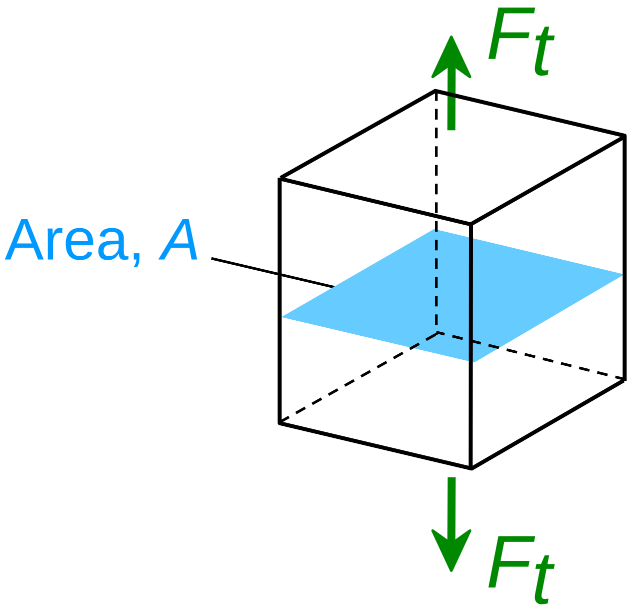
planes
still
sheared

δ_{plastic}

Plastic means permanent!

Engineering Stress

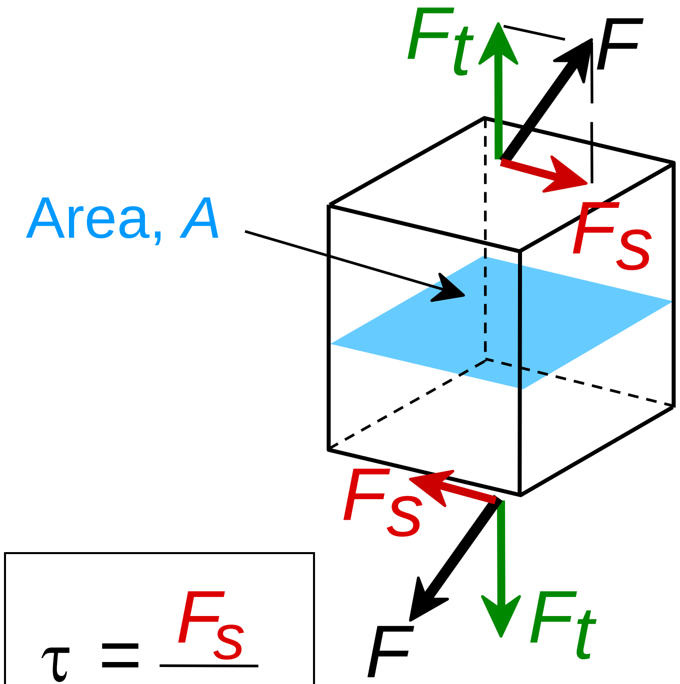
- Tensile stress, σ:



$$\sigma = \frac{F_t}{A_o} = \frac{\mathrm{lb}_f}{\mathrm{in}^2} \text{ or } \frac{\mathrm{N}}{\mathrm{m}^2}$$

original area before loading

- Shear stress, τ:

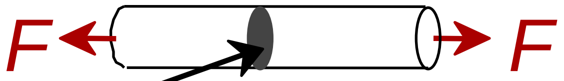


$$\tau = \frac{F_s}{A_o}$$

∴ Stress has units: N/m^2 or lb$_f$/in2

Common States of Stress

- **Simple tension: cable**

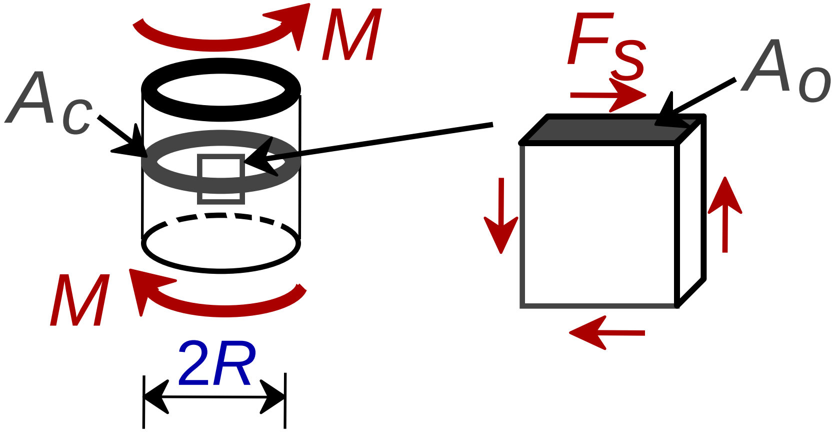


A_o = cross sectional area (when unloaded)

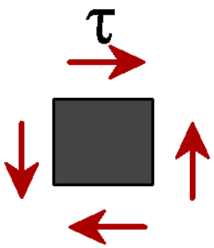
$$\sigma = \frac{F}{A_o}$$

Ski lift (photo courtesy P.M. Anderson)

- **Torsion (a form of shear): drive shaft**



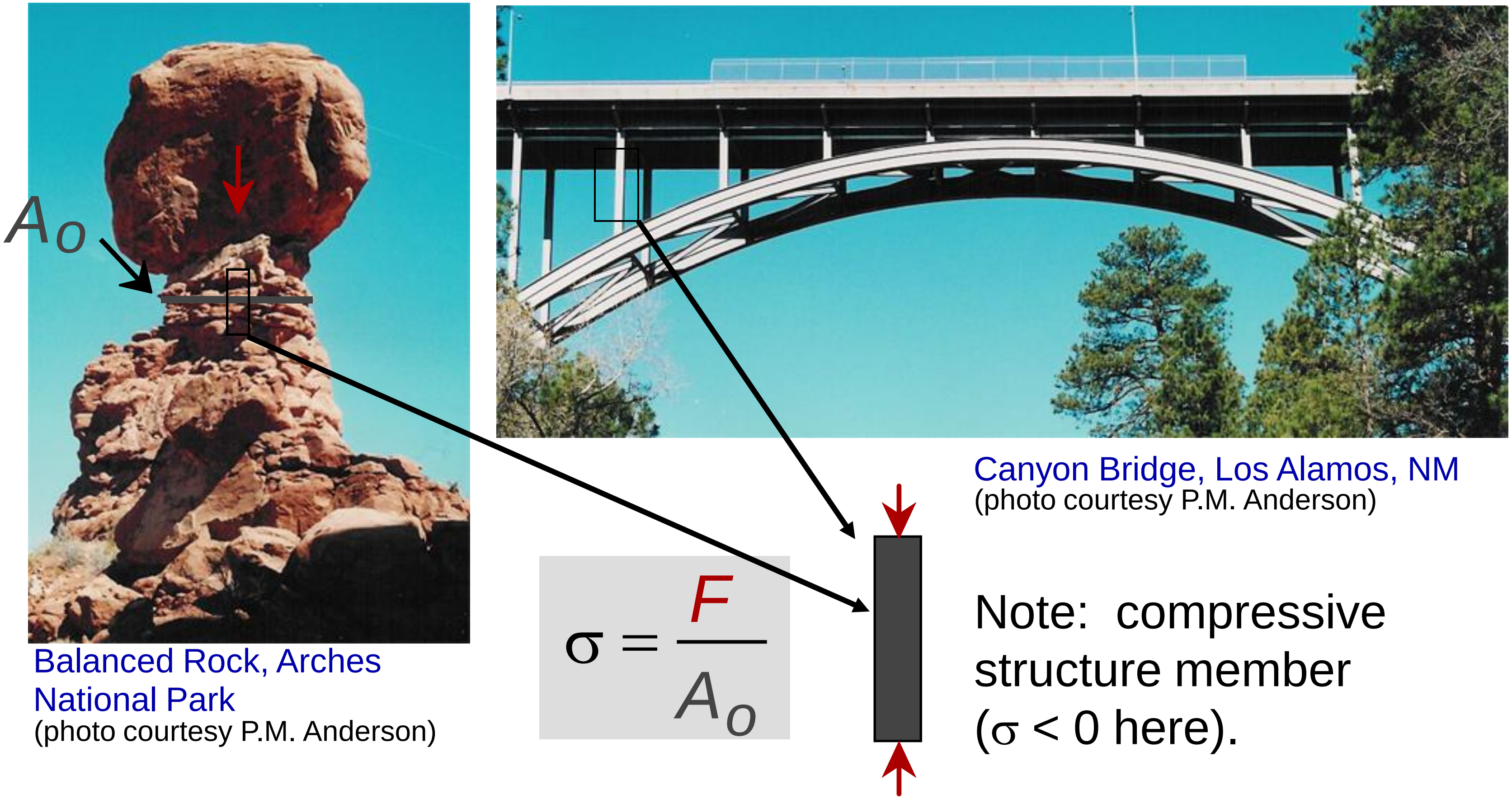
$$\tau = \frac{F_s}{A_o}$$



Note: $\tau = M/A_cR$ here.

OTHER COMMON STRESS STATES (1)

- **Simple compression:**



Balanced Rock, Arches National Park
(photo courtesy P.M. Anderson)

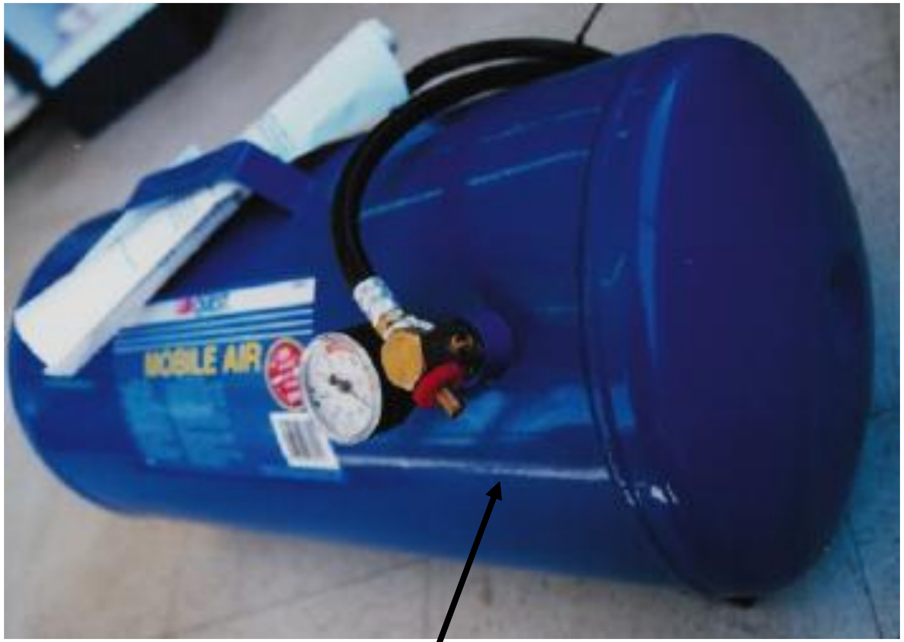
Canyon Bridge, Los Alamos, NM
(photo courtesy P.M. Anderson)

$$\sigma = \frac{F}{A_o}$$

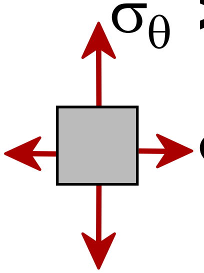
Note: compressive structure member ($\sigma < 0$ here).

OTHER COMMON STRESS STATES (2)

- **Bi-axial tension:**



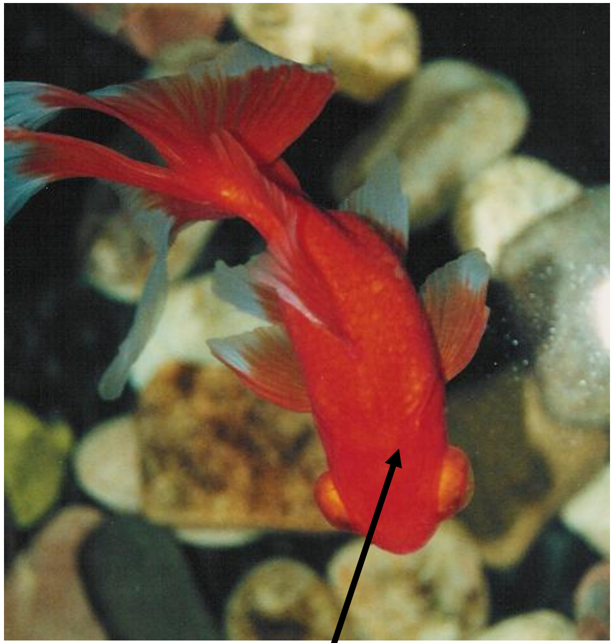
Pressurized tank
(photo courtesy
P.M. Anderson)



$\sigma_\theta > 0$

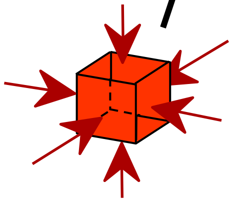
$\sigma_z > 0$

- **Hydrostatic compression:**



Fish under water

(photo courtesy
P.M. Anderson)

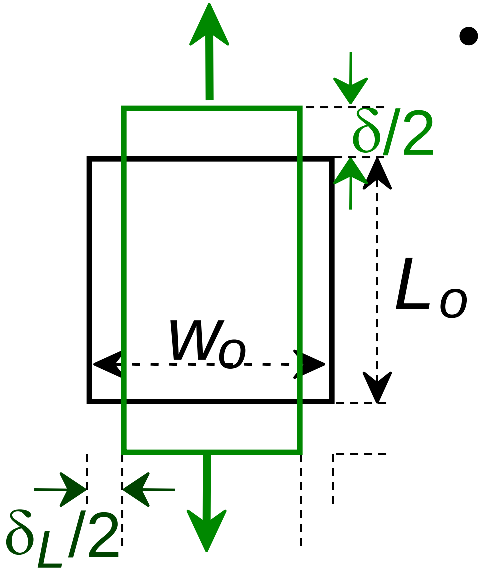


$\sigma_h < 0$

Engineering Strain

- **Tensile** strain:

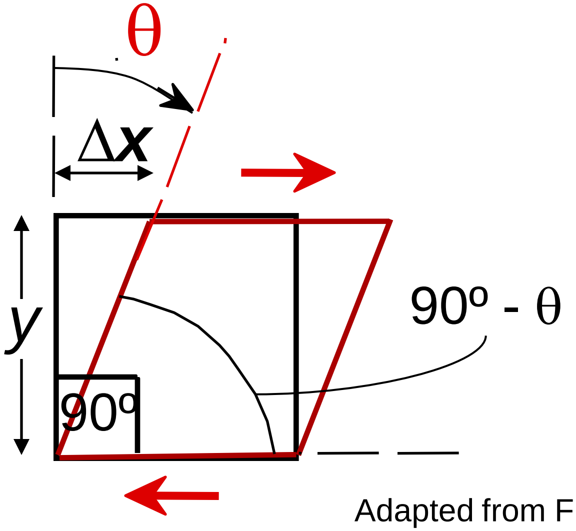
$$\varepsilon = \frac{\delta}{L_o}$$



- **Lateral** strain:

$$\varepsilon_L = \frac{-\delta_L}{w_o}$$

- **Shear** strain:



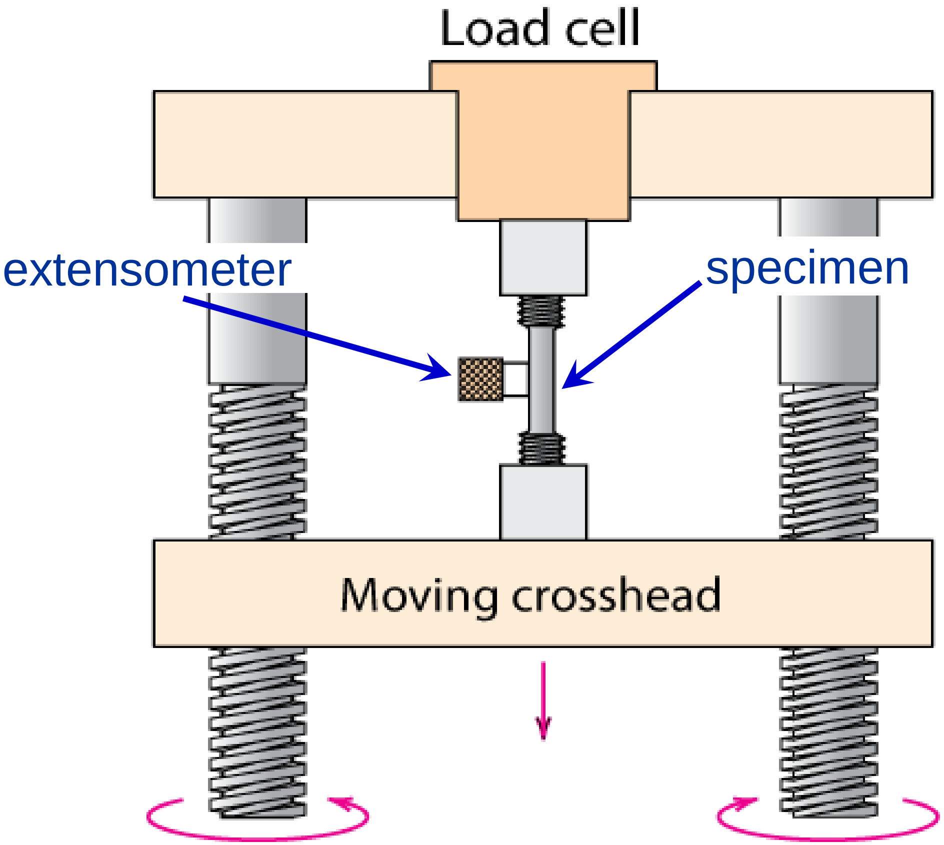
$$\gamma = \Delta x / y = \tan\theta$$

Strain is always dimensionless.

Adapted from Fig. 6.1 (a) and (c), *Callister 7e.*

Stress-Strain Testing

- **Typical tensile test machine**

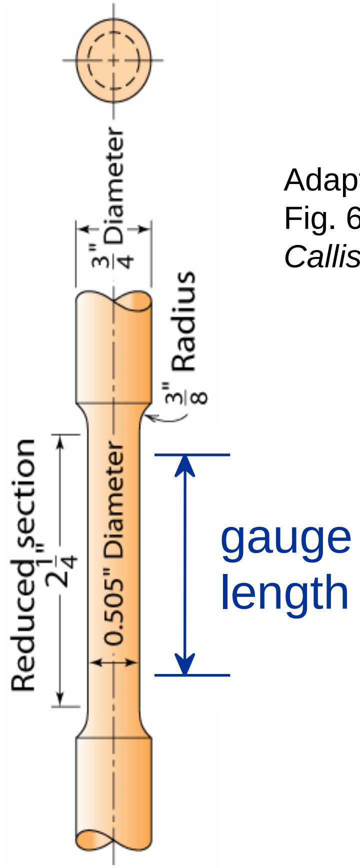


extensometer

specimen

Adapted from Fig. 6.3, *Callister 7e.* (Fig. 6.3 is taken from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, p. 2, John Wiley and Sons, New York, 1965.)

- **Typical tensile specimen**



gauge length

Adapted from Fig. 6.2, *Callister 7e.*

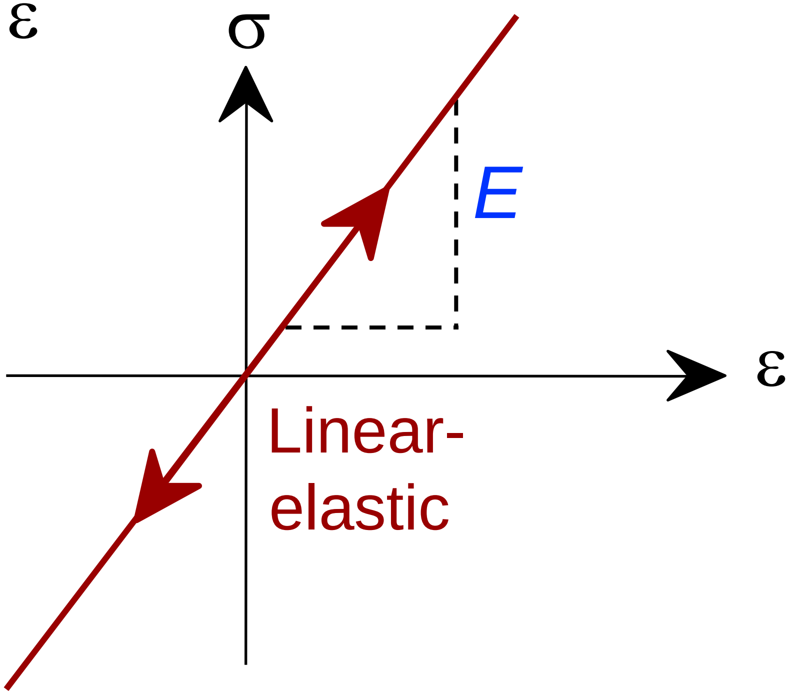
Linear Elastic Properties

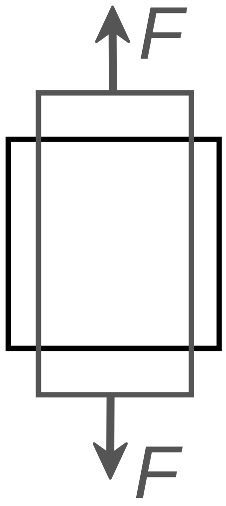
- **Modulus of Elasticity, *E*:**

  (also known as Young's modulus)

- **Hooke's Law:**

$$\sigma = E\,\varepsilon$$



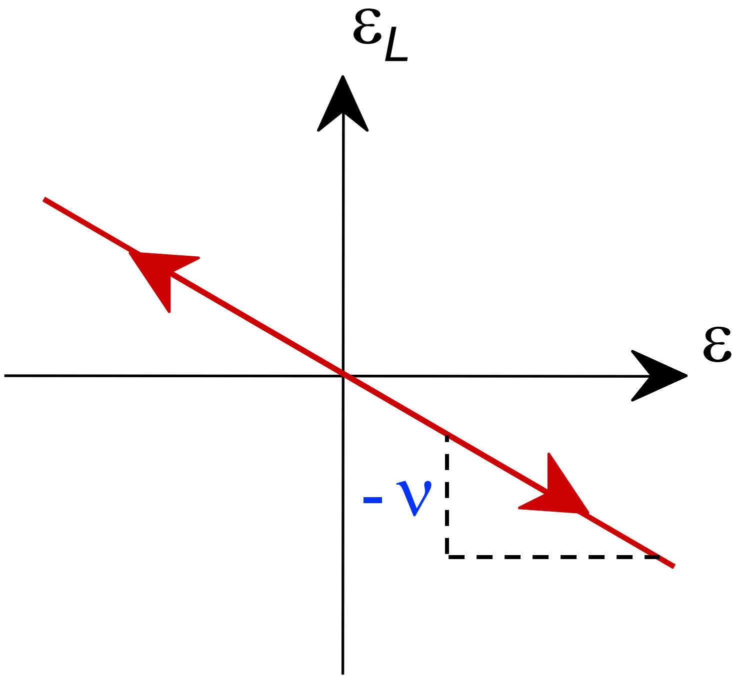


simple
tension
test

Poisson's ratio, ν

- **Poisson's ratio, ν:**

$$\nu = -\frac{\varepsilon_L}{\varepsilon}$$



metals: ν ~ 0.33
ceramics: ν ~ 0.25
polymers: ν ~ 0.40

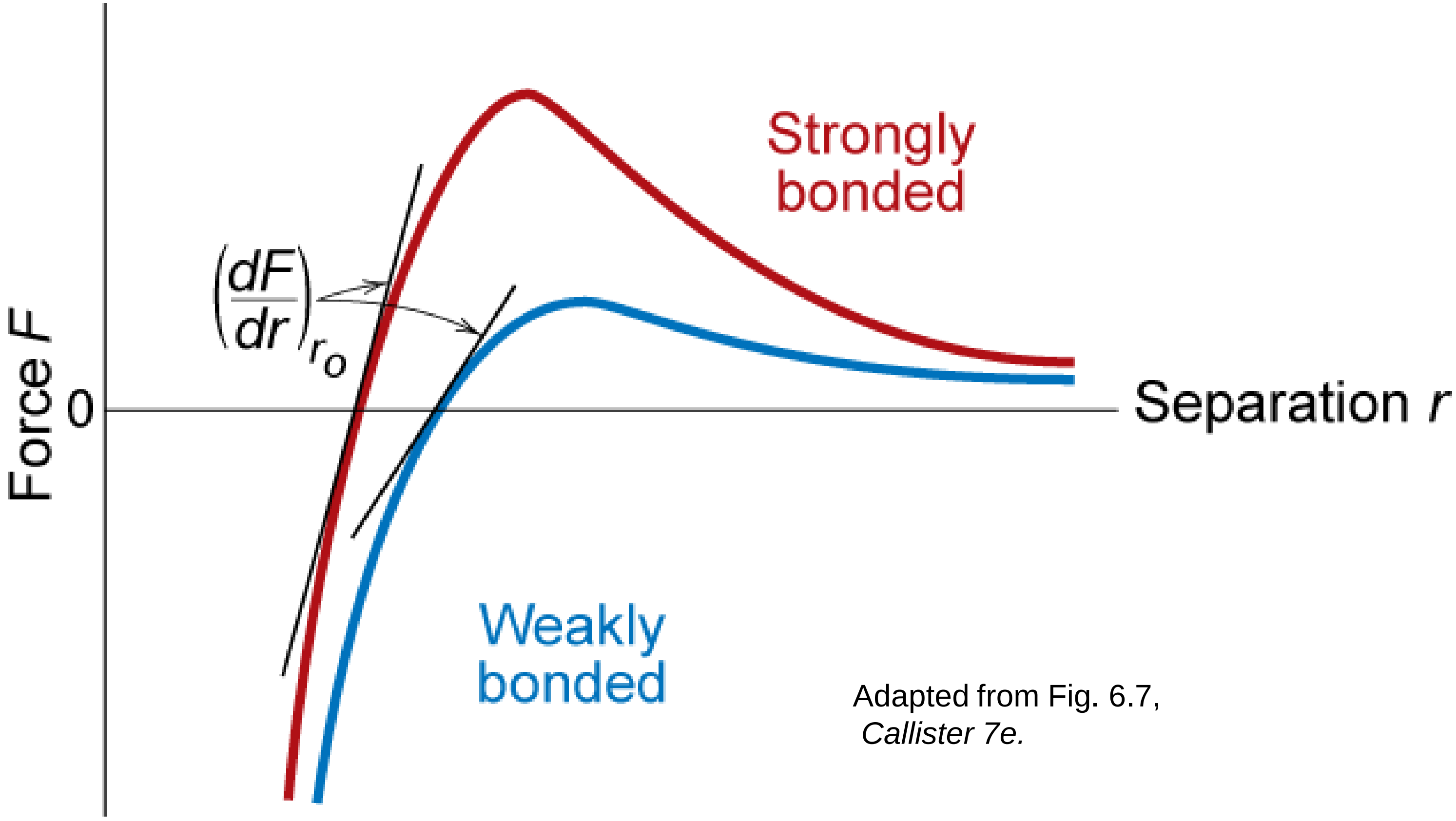
Units:
E: [GPa] or [psi]
ν: dimensionless

–ν > 0.50 density increases

–ν < 0.50 density decreases (voids form)

Mechanical Properties

- Slope of stress strain plot (which is proportional to the elastic modulus) depends on bond strength of metal

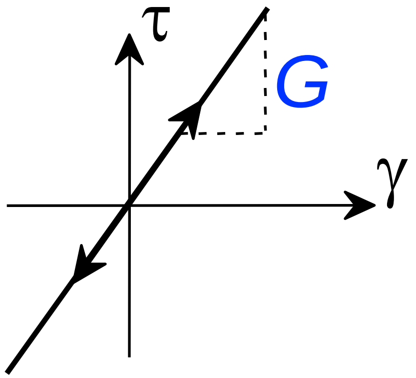


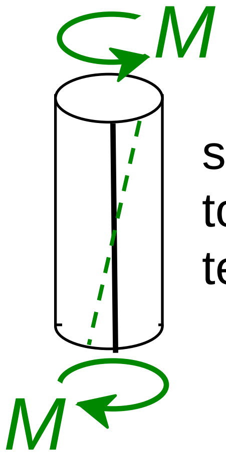
Adapted from Fig. 6.7, *Callister 7e.*

Other Elastic Properties

- **Elastic Shear modulus, *G*:**

$$\tau = G\gamma$$

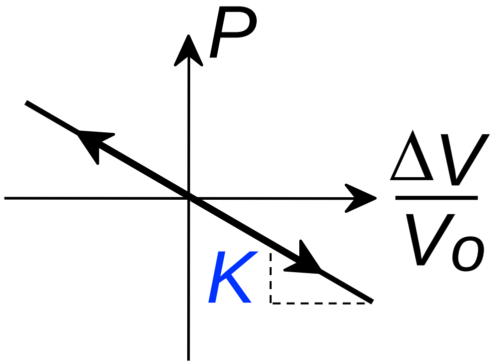


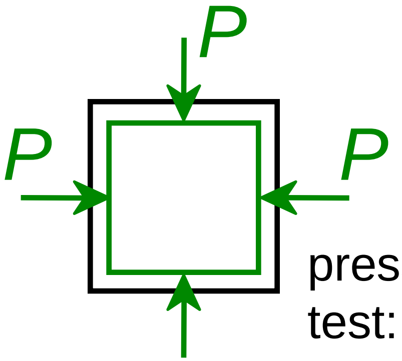


simple
torsion
test

- **Elastic Bulk modulus, K:**

$$P = -K\frac{\Delta V}{V_o}$$



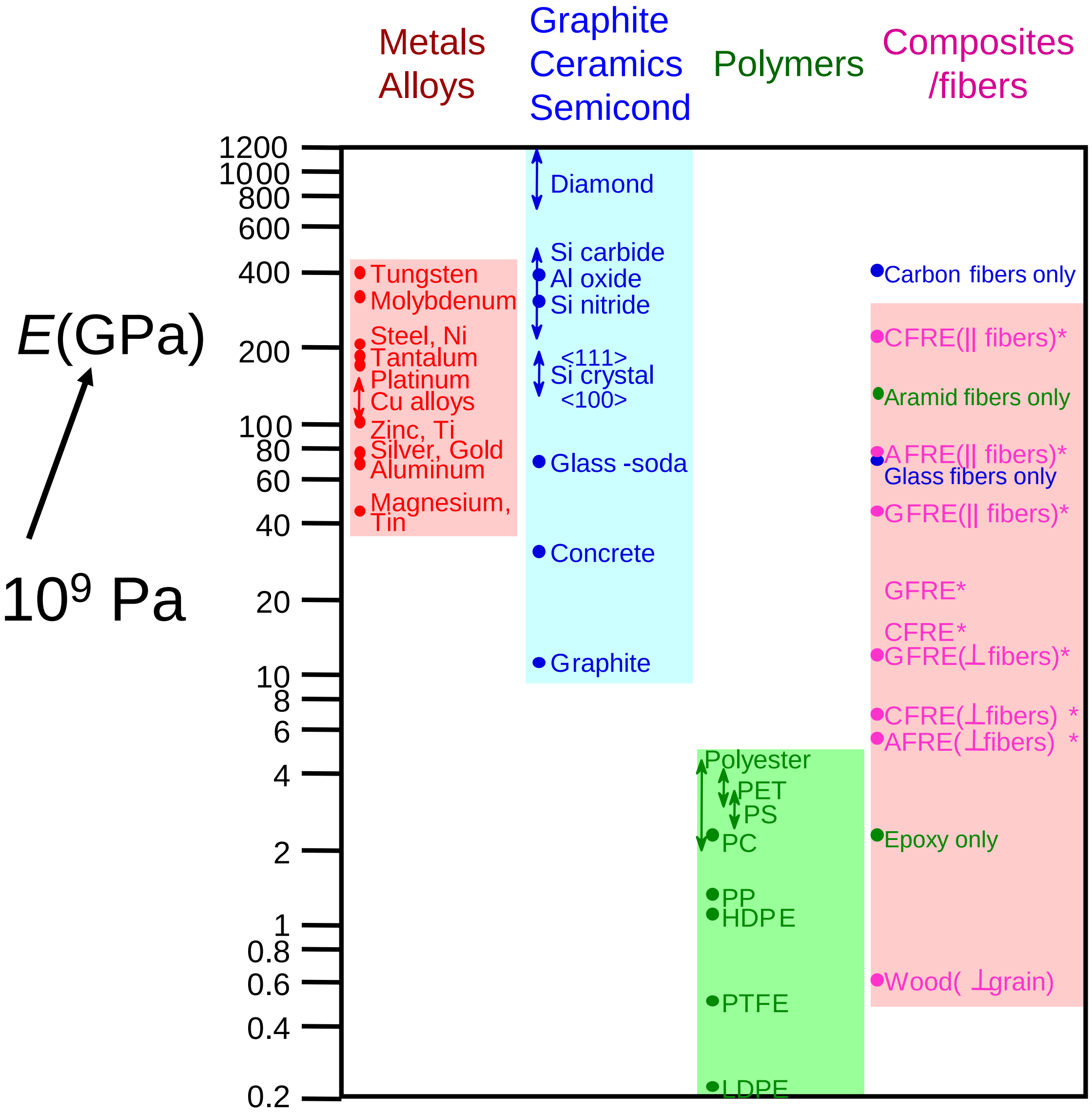


pressure
test: Init.
vol =V_o.
Vol chg.
= ΔV

- **Special relations for isotropic materials:**

$$G = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

Young's Moduli: Comparison



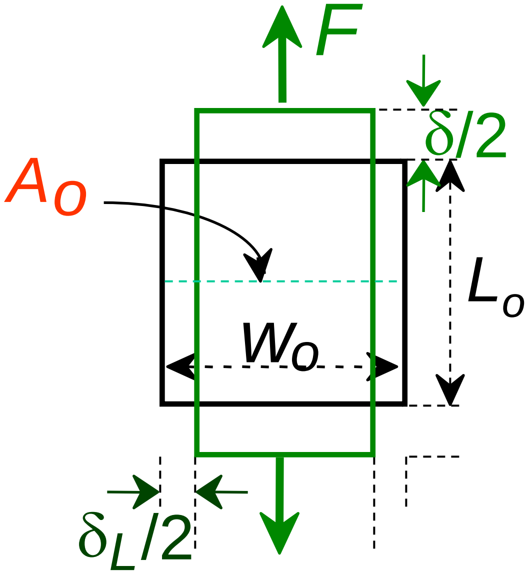
Based on data in Table B2, *Callister 7e*.

Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.

Useful Linear Elastic Relationships

- **Simple tension:**

$$\delta = \frac{F L_o}{E A_o} \qquad \delta_L = -\nu \frac{F w_o}{E A_o}$$

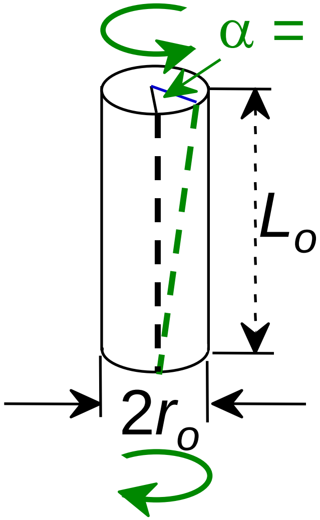


- **Simple torsion:**

$$\alpha = \frac{2 M L_o}{\pi r_o^4 G}$$

M = moment

α = angle of twist

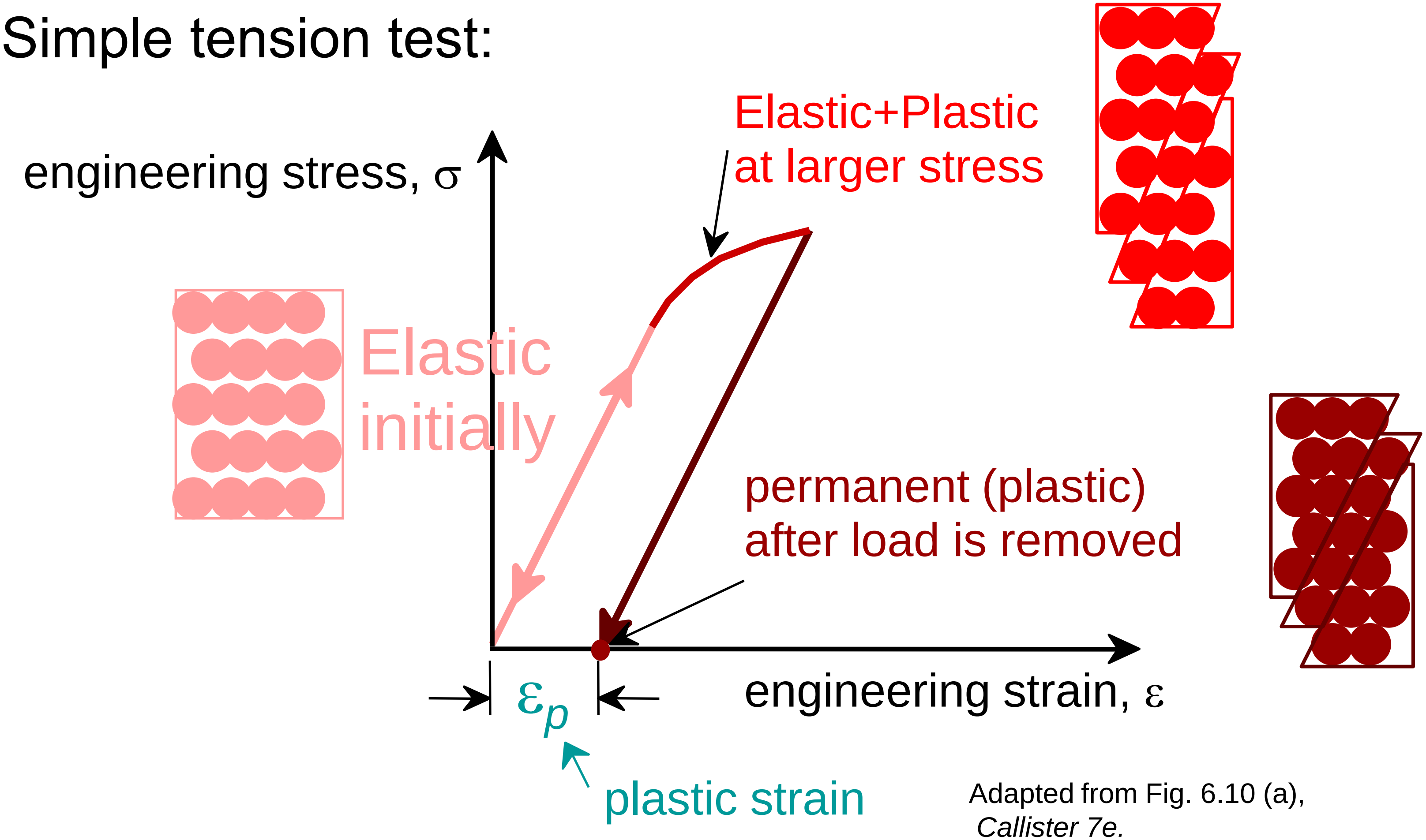


- Material, geometric, and loading parameters all contribute to deflection.
- Larger elastic moduli minimize elastic deflection.

Plastic (Permanent) Deformation

(at lower temperatures, i.e. $T < T_{melt}/3$)

- Simple tension test:



engineering stress, σ

Elastic initially

Elastic+Plastic at larger stress

permanent (plastic) after load is removed

ε_p

plastic strain

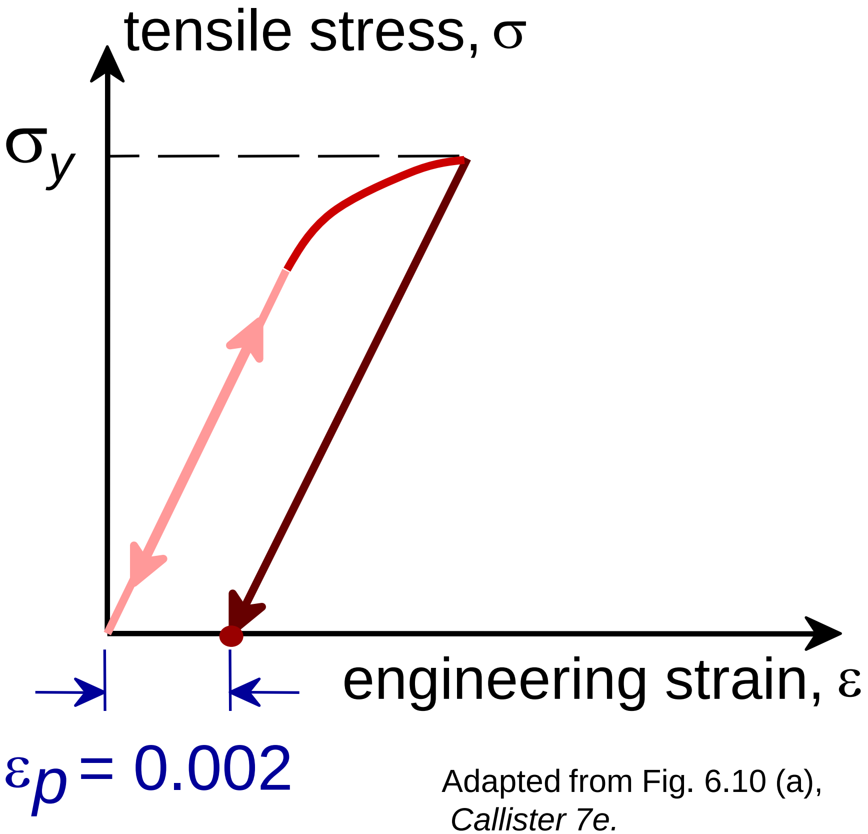
engineering strain, ε

Adapted from Fig. 6.10 (a), *Callister 7e.*

Yield Strength, σ_y

- **Stress at which *noticeable* plastic deformation has occurred.**

when $\varepsilon_p = 0.002$



Adapted from Fig. 6.10 (a), *Callister 7e.*

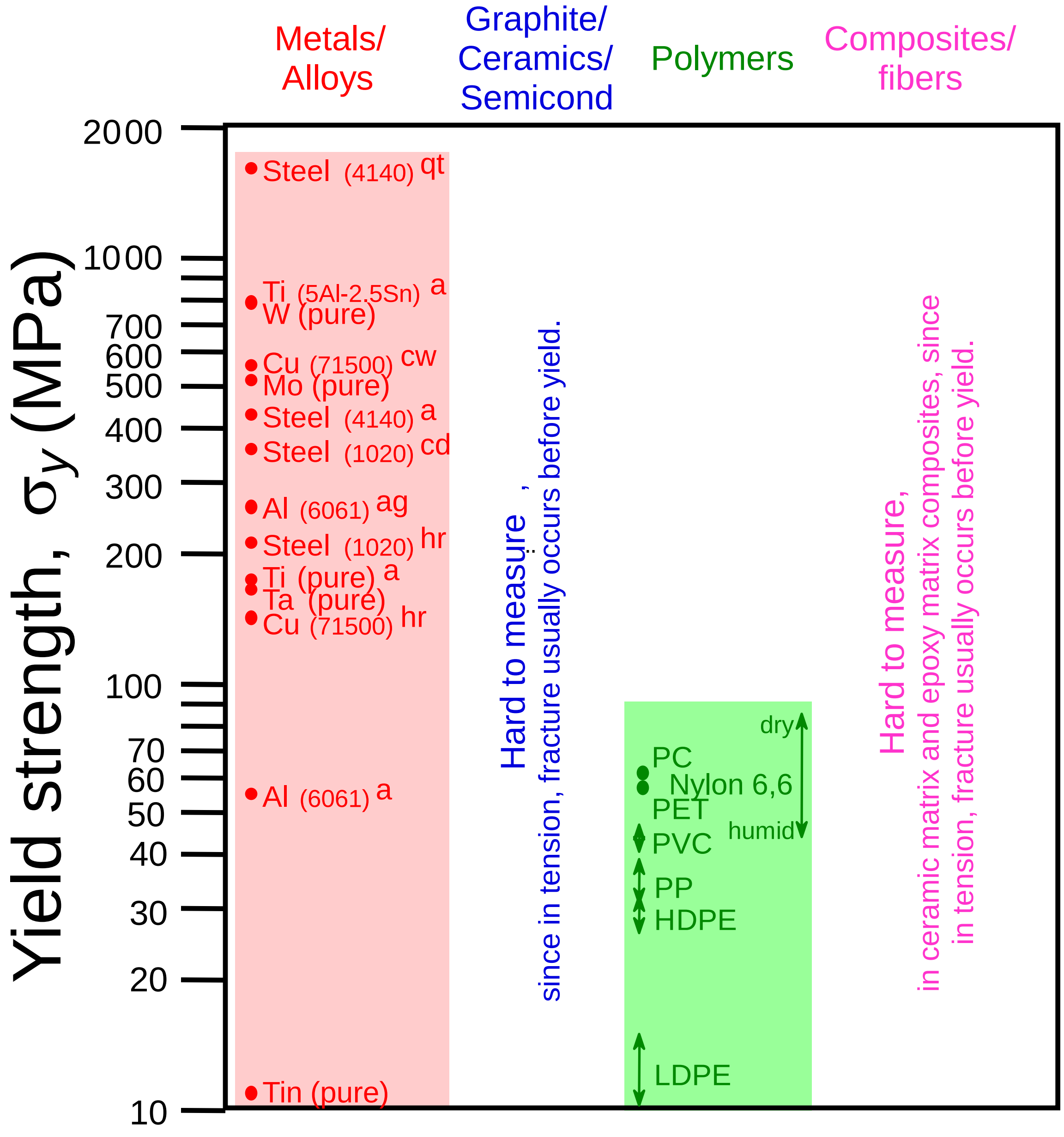
σ_y = yield strength

Note: for 2 inch sample

$\varepsilon = 0.002 = \Delta z/z$

$\therefore\ \Delta z = 0.004$ in

Yield Strength : Comparison



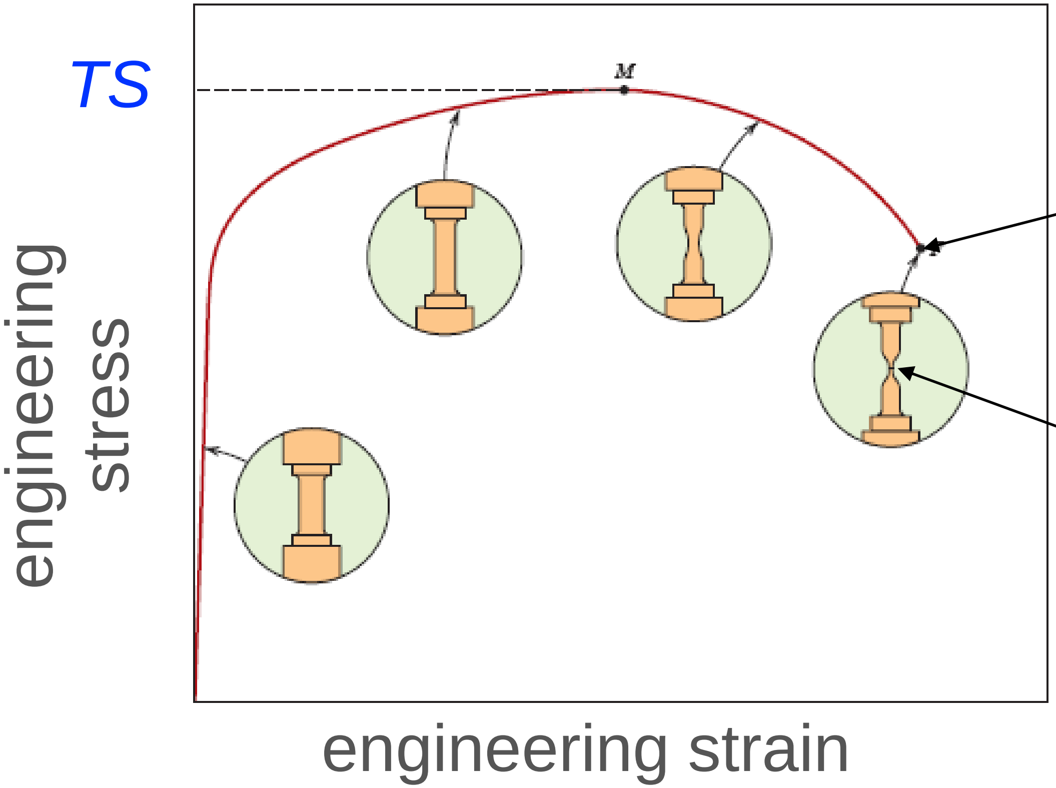
Room *T* values

Based on data in Table B4, *Callister 7e*.

a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

Tensile Strength, TS

- Maximum stress on engineering stress-strain curve.



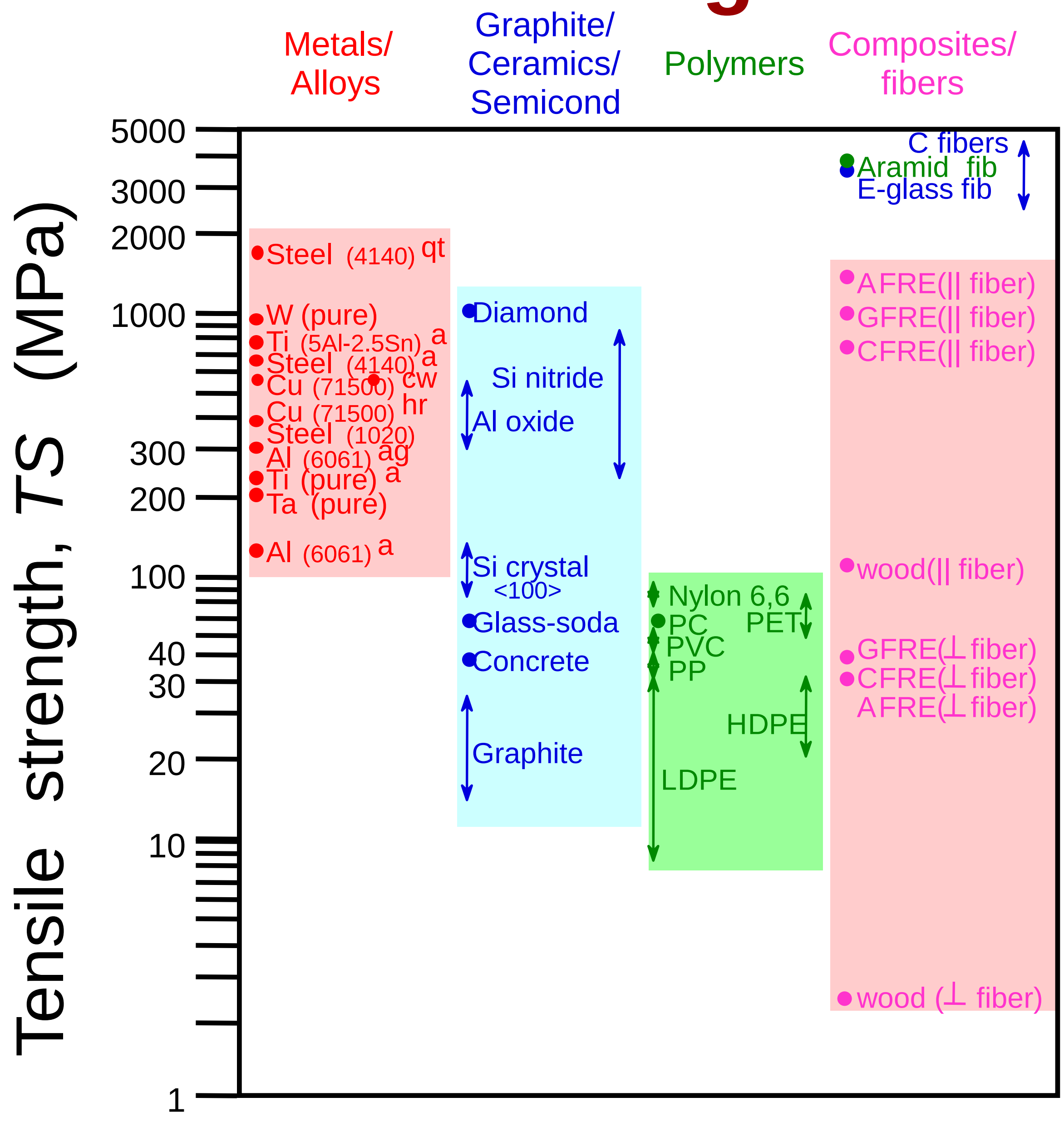
Adapted from Fig. 6.11, *Callister 7e.*

F = fracture or
ultimate
strength

Neck – acts
as stress
concentrator

- Metals: occurs when noticeable necking starts.
- Polymers: occurs when polymer backbone chains are aligned and about to break.

Tensile Strength : Comparison



Room Temp. values

Based on data in Table B4, *Callister 7e*.

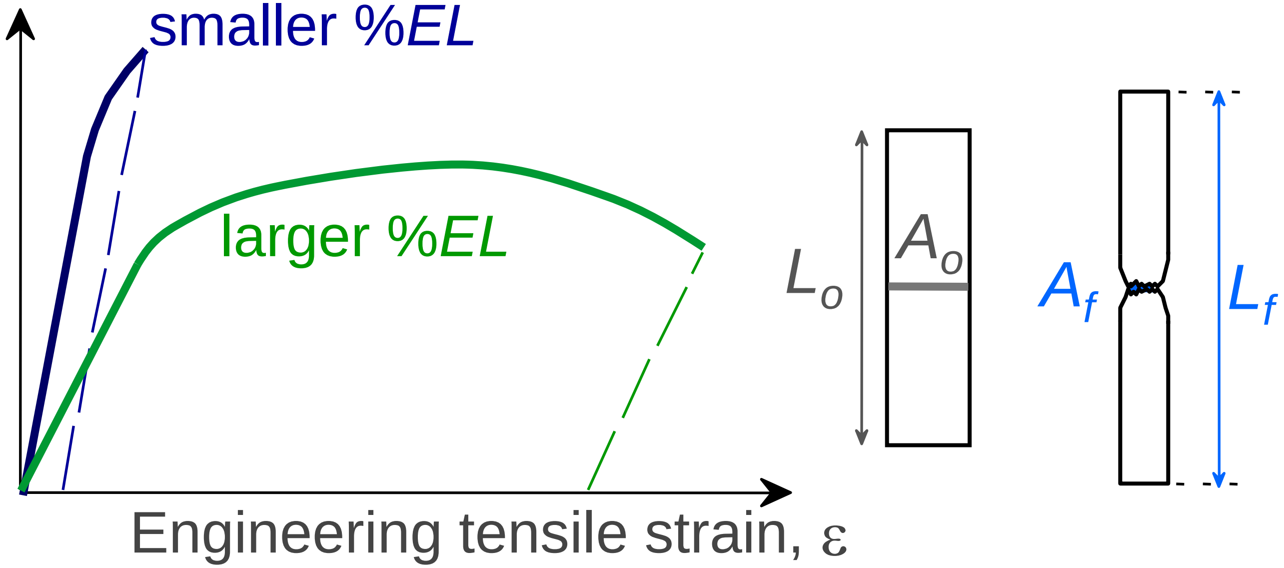
a = annealed
hr = hot rolled
ag = aged
cd = cold drawn
cw = cold worked
qt = quenched & tempered

AFRE, GFRE, & CFRE = aramid, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.

Ductility

- Plastic tensile strain at failure:

$$\%EL = \frac{L_f - L_o}{L_o} \text{ x } 100$$



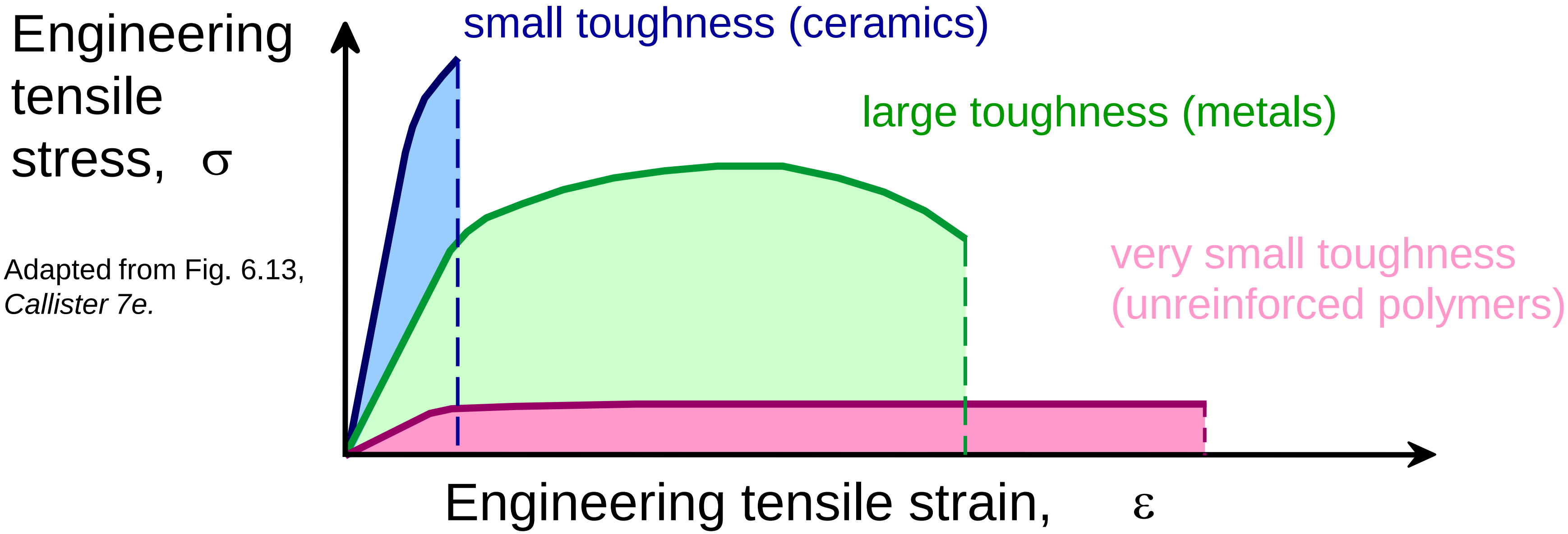
Adapted from Fig. 6.13, *Callister 7e.*

- Another ductility measure:

$$\%RA = \frac{A_o - A_f}{A_o} \text{ x } 100$$

Toughness

- Energy to break a unit volume of material
- Approximate by the area under the stress-strain curve.

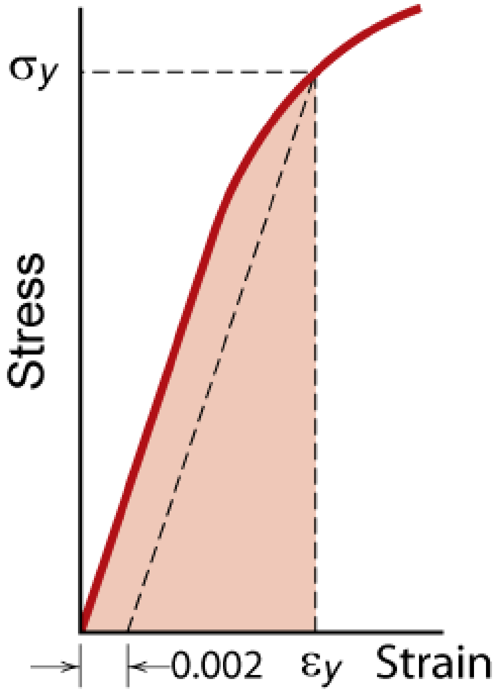


Adapted from Fig. 6.13, *Callister 7e.*

Brittle fracture: elastic energy
Ductile fracture: elastic + plastic energy

Resilience, U_r

- Ability of a material to store energy
  - Energy stored best in elastic region



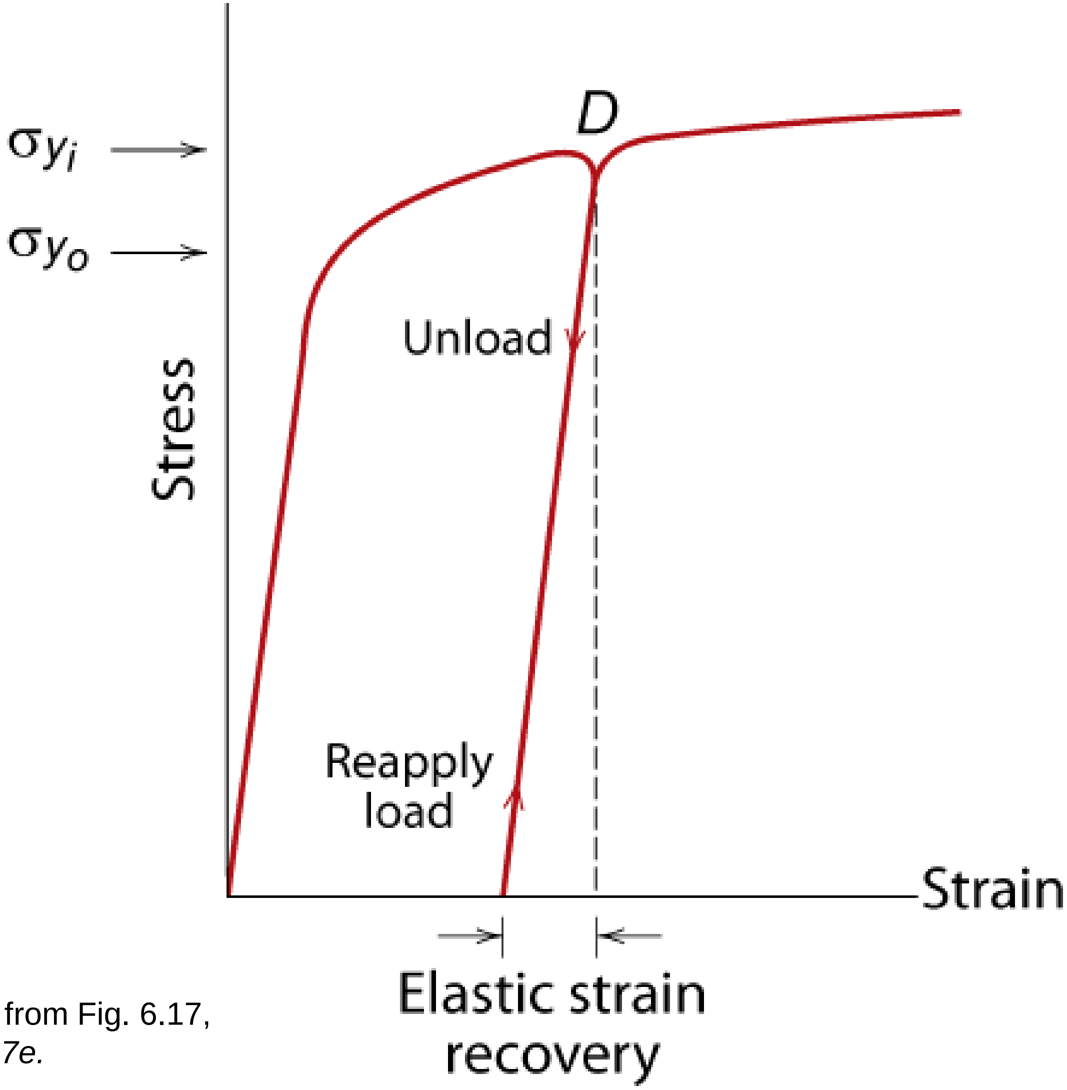
Adapted from Fig. 6.15, *Callister 7e.*

$$U_r = \int_0^{\varepsilon_y} \sigma d\varepsilon$$

If we assume a linear stress-strain curve this simplifies to

$$U_r \cong \frac{1}{2} \sigma_y \varepsilon_y$$

Elastic Strain Recovery



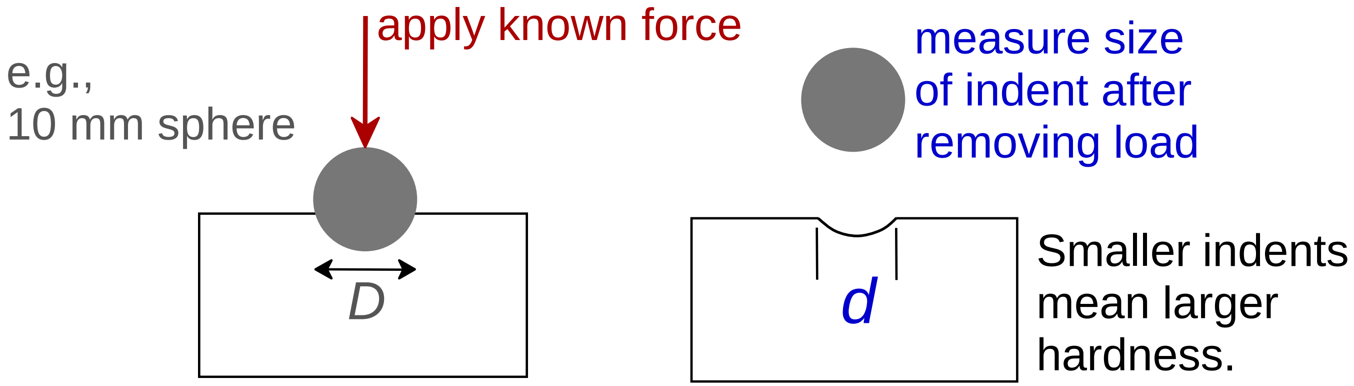
Adapted from Fig. 6.17, *Callister 7e.*

Hardness

- Resistance to permanently indenting the surface.
- Large hardness means:

--resistance to plastic deformation or cracking in compression.

--better wear properties.



e.g., 10 mm sphere

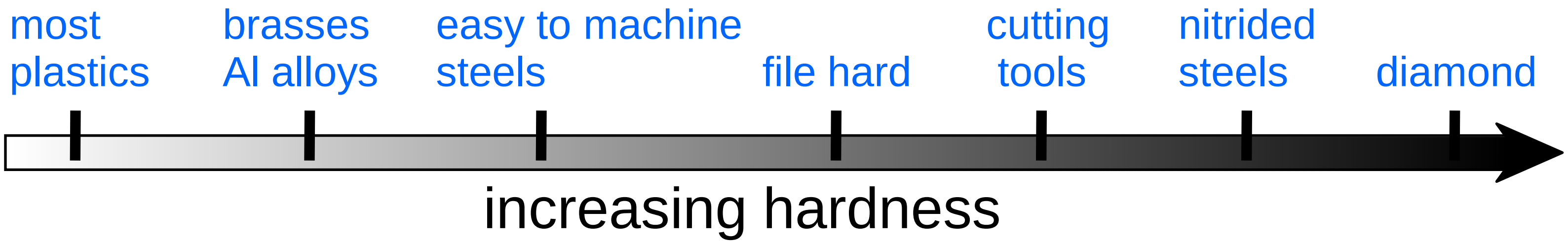
apply known force

D

measure size of indent after removing load

d

Smaller indents mean larger hardness.



most plastics

brasses Al alloys

easy to machine steels

file hard

cutting tools

nitrided steels

diamond

increasing hardness

Hardness: Measurement

- Rockwell
  - – No major sample damage
  - – Each scale runs to 130 but only useful in range 20-100.
  - – Minor load 10 kg
  - – Major load 60 (A), 100 (B) & 150 (C) kg
    - A = diamond, B = 1/16 in. ball, C = diamond

- HB = Brinell Hardness
  - – *TS* (psia) = 500 x HB
  - – *TS* (MPa) = 3.45 x HB

Hardness: Measurement

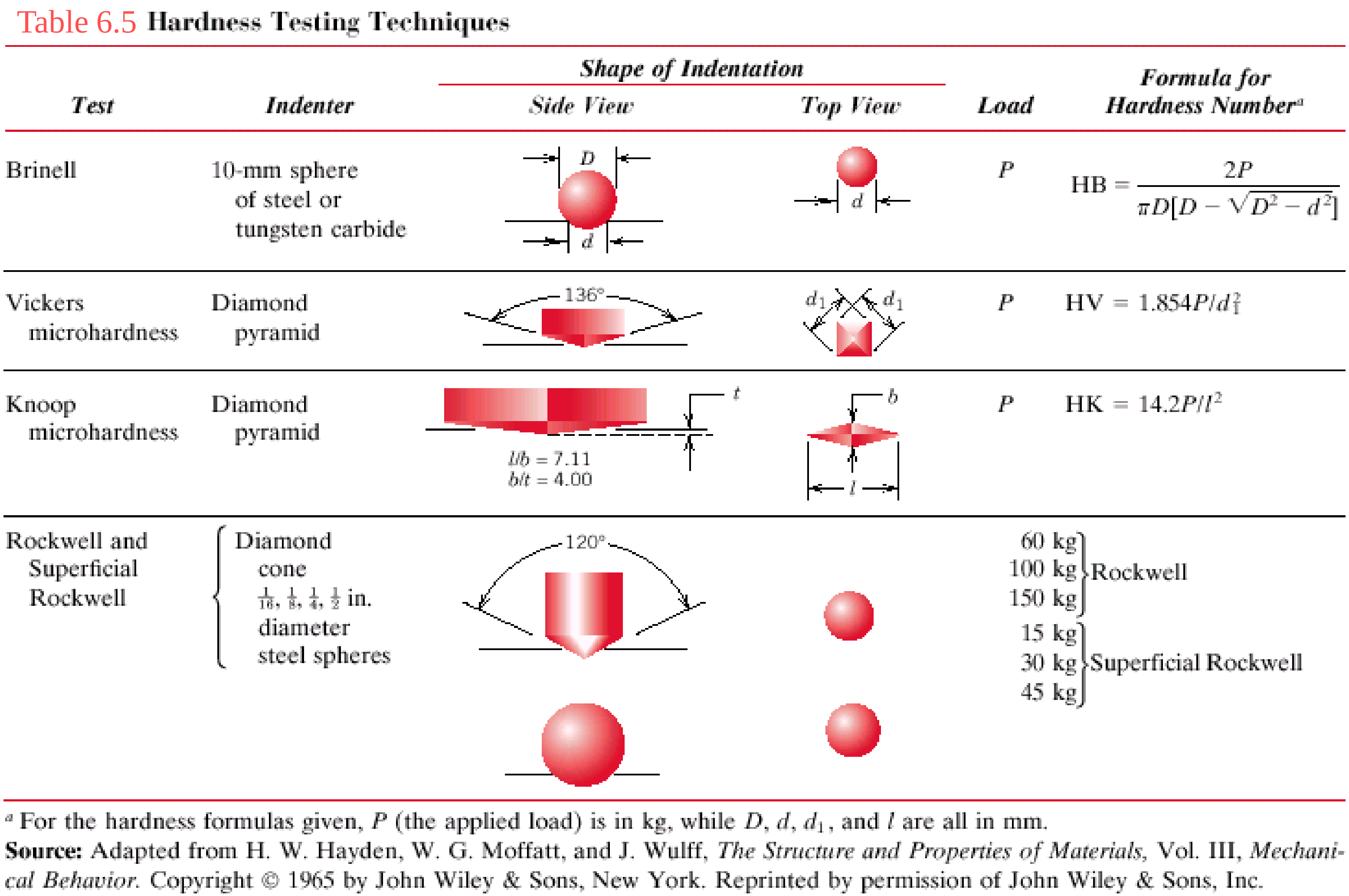


Table 6.5 **Hardness Testing Techniques**

Test	*Indenter*	*Shape of Indentation*: *Side View*	*Shape of Indentation*: *Top View*	*Load*	*Formula for Hardness Number*[a]
Brinell	10-mm sphere of steel or tungsten carbide			P	$HB = \dfrac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			P	$HV = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid	$l/b = 7.11$ $b/t = 4.00$		P	$HK = 14.2P/l^2$
Rockwell and Superficial Rockwell	Diamond cone $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ in. diameter steel spheres			60 kg, 100 kg, 150 kg: Rockwell 15 kg, 30 kg, 45 kg: Superficial Rockwell	

[a] For the hardness formulas given, P (the applied load) is in kg, while D, d, d_1, and l are all in mm.

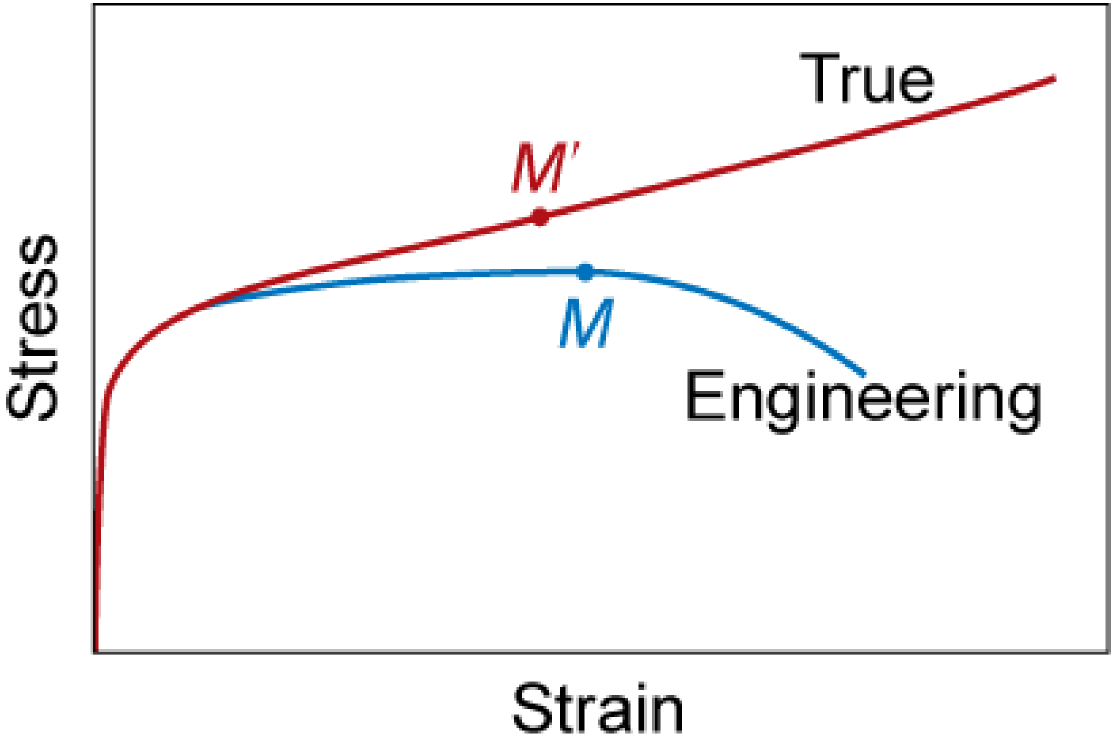
Source: Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*. Copyright © 1965 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.

True Stress & Strain

Note: S.A. changes when sample stretched

- True stress $\sigma_T = F/A_i$
- True Strain $\varepsilon_T = \ln(\ell_i/\ell_o)$

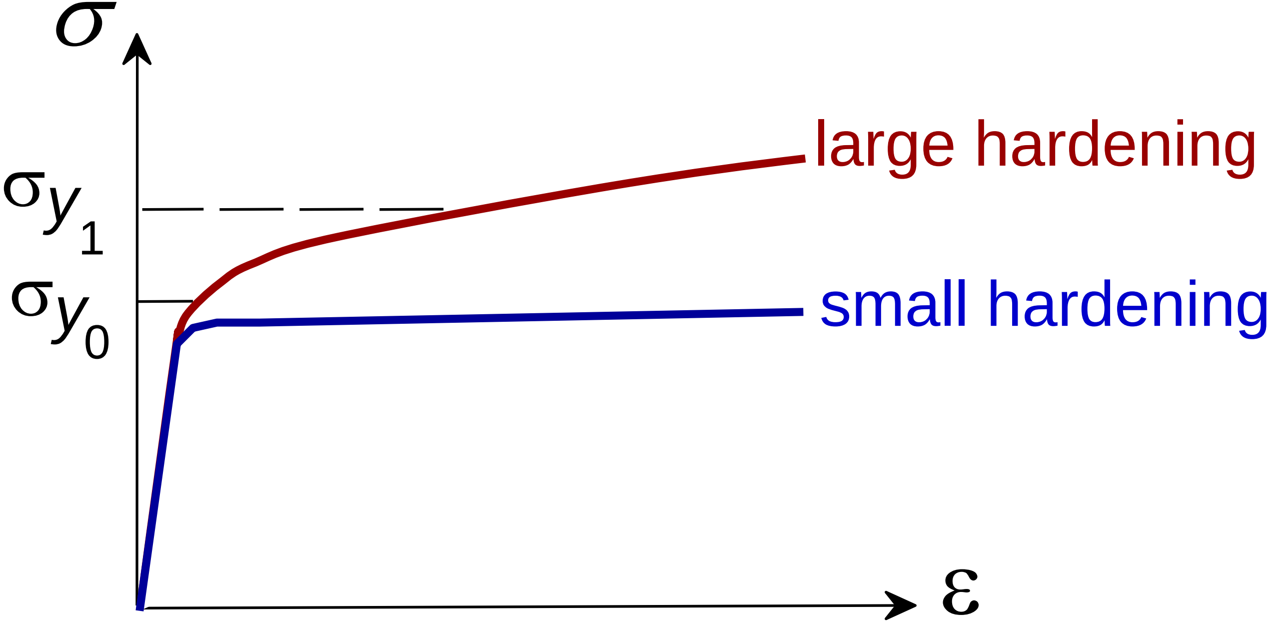
$$\boxed{\begin{aligned}\sigma_T &= \sigma(1+\varepsilon)\\ \varepsilon_T &= \ln(1+\varepsilon)\end{aligned}}$$



Adapted from Fig. 6.16, *Callister 7e.*

Hardening

- An increase in σ_y due to plastic deformation.



- Curve fit to the stress-strain response:

$$\sigma_T = K\left(\varepsilon_T\right)^n$$

hardening exponent:
n = 0.15 (some steels)
to n = 0.5 (some coppers)

“true” stress (F/A)

“true” strain: $\ln(L/L_o)$

Variability in Material Properties

- Elastic modulus is material property
- Critical properties depend largely on sample flaws (defects, etc.). Large sample to sample variability.
- Statistics
  - Mean

$$\bar{x} = \frac{\overset{n}{\Sigma} x_n}{n}$$

  - Standard Deviation

$$s = \left[\frac{\overset{n}{\Sigma}\left(x_i - \bar{x}\right)^2}{n-1}\right]^{\frac{1}{2}}$$

where *n* is the number of data points

Design or Safety Factors

- Design uncertainties mean we do not push the limit.
- Factor of safety, *N*

$$\sigma_{working} = \frac{\sigma_y}{N}$$

Often *N* is between 1.2 and 4

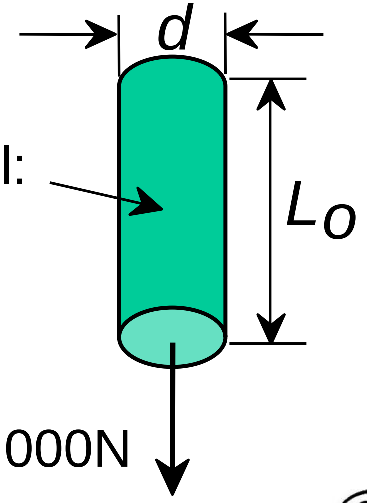
- Example: Calculate a diameter, *d*, to ensure that yield does not occur in the 1045 carbon steel rod below. Use a factor of safety of 5.

$$\sigma_{working} = \frac{\sigma_y}{N}$$

$$\frac{220{,}000\,N}{\pi\left(d^2/4\right)}$$

5

1045 plain carbon steel:
$\sigma_y = 310$ MPa
TS = 565 MPa



F = 220,000N

$$\boxed{d = 0.067 \text{ m} = 6.7 \text{ cm}}$$

Summary

- Stress and strain: These are size-independent measures of load and displacement, respectively.
- Elastic behavior: This reversible behavior often shows a linear relation between stress and strain. To minimize deformation, select a material with a large elastic modulus (E or G).
- Plastic behavior: This permanent deformation behavior occurs when the tensile (or compressive) uniaxial stress reaches σ_y.
- Toughness: The energy needed to break a unit volume of material.
- Ductility: The plastic strain at failure.

ANNOUNCEMENTS

Reading:

Core Problems:

Self-help Problems: